SHIVALIK SR. SEC. SCHOOL, BHARTHARI ROAD, BEHROR

CLASS--XI

UNIT:- SEQUENCE AND SERIES → AP, GP & HP

SUBJECT- Mathematics (Ramesh Suthar Sir)

- **Sequence :-** A sequence is a function whose domain is the set N of natural numbers.
  - Its denoted by $<a_n>$ or $\{a_n\}$.
  - Fibonacci sequence is given by $a_1 = 1$, $a_2 = 1$ and $a_{n+1} = a_{n+} a_{n-1}$; $n \geq 2$. The term of this sequence are 1, 1, 2, 3, 5, 8 ....
- **Series :-** If $a_1, a_2, a_3, a_4, a_5, \ldots\ldots, a_n, \ldots\ldots$ is a sequence, then the expression

  $a_1 + a_2 + a_3 + a_4 + \ldots\ldots a_n + \ldots\ldots$ is a series.
- **Progressions :-** Those sequence whose terms follow certain patterns are called progressions.
- **Arithmetic Progression :-** A sequence is called an arithmetic progression if the difference of a term is always same.

  i.e. Common difference (d) = $a_{n+1} - a_n$
- **General form of an A.P.** :-Let a be the first term and d be the common difference of A.P., then general form is :-

  $a, a + d, a + 2d, a + 3d, \ldots\ldots, a + (n - 1)d, \ldots\ldots$

  $\therefore d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots = a_n - a_{n-1} = a_{n+1} - a_n = \ldots\ldots$.
  - nth term of an A.P.

    $a_n = a + (n - 1)d$, $L = a + (n - 1)d$
  - ⇨ nth term from the end

    $a_n = L - (n-1)d$
  - ⇨ nth term from the end

    $a_{m-n+1} = a + (m - n)d$
- **Terms of An A.P.** :-

No. Of terms	Terms
2	a – d, a + d
3	a – d, a, a + d
4	a – 3d, a – d, a + d, a + 3d
5	a – 2d, a – d, a, a + d, a + 2d
6	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d

- **Sum to n terms on A.P.** :-

  The sum of n terms of an A.P. with first term 'a' and common difference 'd' is given by.

  $S_n = \frac{n}{2}[2a + (n - 1)d]$

⇨ $S_n = \frac{n}{2}[a + \{a + (n-1)d\}]$

⇨ $S_n = \frac{n}{2}(a + L)$

- If the sum S_n of n terms of a sequence is given, then nth term $a_n = S_n - S_{n-1}$
- Three numbers a, b, c are in A.P. iff $2b = a + c$.

Arithmetic Mean :-

(i) If A is an arithmetic mean between a & b. Then a, A, b in A.P.

So, $A = \frac{a+b}{2}$

(ii) If $A_1, A_2, A_3, \ldots\ldots$, An be n arithmetic means between two quantities a and b. Then a, $A_1, A_2, \ldots\ldots, A_{n-1}, A_n$, b in A.P.

Then $A_n = a + [\frac{n(b-a)}{n+1}]$

Here :- $d = \frac{(b-a)}{(n+1)}$

Geometric progression :-

A sequence a1, a2, a3, ..., an, ... is called a geometric progression if

$\frac{a_{n+1}}{a_n}$ = constant, ∀ n ϵ N.

General form of a G.P. :- If a be the first term and r be the common ratio of G.P. , is :-

a, ar, ar^2, ar^3, ..., ar^{n-1}, ...

- nth term of G.P. an = ar^{n-1}.
- last term of G.P. $L = ar^{n-1}$.
- Common ratio = $\frac{a_{n+1}}{a_n} = r = \frac{a_2}{a_1} = \frac{a_3}{a_2} =$ ....

Terms of a G.P. :-

No. Of terms	Terms
2	$\frac{a}{r}$, ar
3	$\frac{a}{r}$, a, ar
4	$\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3
5	$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2
6	$\frac{a}{r^5}, \frac{a}{r^3}, \frac{a}{r}$, ar, ar^3, ar^5

Sum of n terms of a G.P. :-The sum of n terms of a G.P. with first term 'a' and common ratio 'r' is given by.

$Sn = \frac{a(1-r^n)}{(r-1)}$; r > 1

$$S_n = \frac{a\,(1-r^n)}{(r-1)}\ ;\ r < 1$$

- **<u>Sum of an infinite G.P.</u>** :-

  The sum of an infinite G.P. with first term 'a' and common ratio 'r' is given by

  $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$, ($|r| < 1$)

- **<u>Geometric Mean</u>** :-

  (i) If G be a geometric mean between a & b, then

  a, G, b in G.P.

  ⇨ $G^2 = ab$

  ⇨ $G = \sqrt{ab}$

  (ii) If $G_1, G_2, \ldots\ldots, G_n$ be n geometric mean between a & b, then

  a , $G_1, G_2, \ldots\ldots, G_n$, b in G.P.

  Common Ratio $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

  $G_n = ar^n = a\left[\left(\frac{b}{a}\right)^{\frac{1}{n+1}}\right]^n$

  $G_n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

  - ❖ If A and G are respectively arithmetic and geometric means between two positive number a and b, then A > G
  - ❖ If A and G be the arithmetic mean and geometric mean between two positive numbers, then the numbers are $A \pm \sqrt{A^2 + G^2}$

- **<u>Arithmetic Geometric sequence :-</u>** $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ and $b_1, b_2, b_3, \ldots\ldots, b_n, \ldots\ldots$ is a G.P., then the sequence $a_1b_1, a_2b_2, a_3b_3 \ldots\ldots, a_nb_n, \ldots\ldots$ is said to be an arithmetic – geometric sequence.

  a, (a+d)r, (a+2d)r^2, (a+3d)r^3, ……………. .

  - ❖ nth term of AP – GP sequence $[a + (n-1)d]\, r^{n-1}$
  - ❖ Sum of n terms of an arithmetic geometric sequence

  $$S_n = \begin{cases} \frac{a}{(1-r)} + \frac{dr\,(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]\,r^n}{(1-r)} & ; r \neq 1 \\ \frac{n}{2}\,[\,2a + (n-1)d] & ; r = 1 \end{cases}$$

  - ❖ Sum of an infinite arithmetic – geometric sequence

  $$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

- **Sum of first n natural numbers :-**

  $\sum n = 1 + 2 + 3 + \text{..........} + n = \frac{n(n+1)}{2}$

- **Sum of the squares of first n natural numbers :-**

  $\sum n^2 = 1^2 + 2^2 + 3^2 + \text{..............} + n^2 = \frac{n(n+1)(2n+1)}{6}$

- **Sum of the cubes of first n natural numbers :-**

  $\sum n^3 = 1^3 + 2^3 + 3^3 + \text{..............} + n^3 = [\frac{n(n+1)}{2}]^2$

- **HARMONIC PROGRESSION:-** A sequence a_1, a_2, a_3, ... a_n, ... of non zero numbers is called a Harmonic progression if sequence.

  $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \text{.................}, \frac{1}{a_n}, \text{............}$ is on A.P.

  $\therefore d = \frac{1}{a_2} - \frac{1}{a_1}$; $a_n = \frac{1}{a+(n-1)d}$; $a = \frac{1}{a_1}$

  - Harmonic mean between two number a & b. $H = \frac{2ab}{a+b}$
  - nth Harmonic means between a & b.

    a, H_1, H_2, ... H_n, b in H.P.

    ⇨ $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \text{..............}, \frac{1}{H_n}$, b in H.P.

    ⇨ $d = \frac{a-b}{(n+a)ab}$

    ⇨ $\frac{1}{H_n} = \frac{1}{a} + nd = \frac{1}{a} + \frac{n(a-b)}{(n+1)ab}$

    ⇨ $\frac{1}{H_n} = \frac{bn+b+an-bn}{ab\,(n+1)} = \frac{b+an}{ab\,(n+1)}$

    ⇨ $H_n = \frac{ab\,(n+1)}{an+b}$

    ⇨ $A \geq G \geq H$

    ⇨ $AM \geq GM \geq HM$

  - If A, G, H in G.P., then $G^2 = AH$